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Research Article

The Valerian Operator: Reversal Dynamics and Convex Solutions in Lucas-Type Differential Equations

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Abstract:
This paper investigates the existence and uniqueness of strictly convex solutions to a novel class of boundary value problems driven by the Valerian Reversal Operator. By introducing a temporal shift mechanism analogous to sequence reversal, we demonstrate that the eigenvalues of such systems are intrinsically linked to the Lucas number sequence. Furthermore, we establish the $\mathcal{L}_m$ (Limit-Monotone) stability criterion, showing that reversing the dynamic flow yields exact survivor formulas for the differential states.

Keywords: Differential Equations, Convex Functions, Lucas Sequence, Valerian Operator, Reversal Dynamics
Mathematics Subject Classification: Primary 34B15; Secondary 11B39, 26A51

1. Introduction

The study of differential equations with deviating arguments usually focuses on delays or advances in time. However, recent developments in combinatorial survival formulas, such as the Talisman Shuffle, suggest that a complete "reversal" of the chronological sequence can yield highly symmetric and predictable mathematical behaviors.

In this paper, we construct a continuous analogue to these discrete reversal mechanics. We introduce the *Valerian Operator* $\mathcal{V}$, which acts on a function space by reflecting the temporal variable across a designated midpoint. When combined with the structural properties of strictly convex functions, this operator generates a unique differential topology.

2. The Valerian Operator and the $\mathcal{L}_m$ Constant

Definition 1. *Let $f \in C^2([0,T],\mathbb{R})$ be a strictly convex function. The Valerian Reversal Operator $\mathcal{V}$ is defined as:*

$$\mathcal{V}[f](t) = f(T-t) - \int_0^t f''(s) L_{\lfloor s \rfloor} ds$$

where L_k denotes the k-th term of the Lucas number sequence ($L_0 = 2, L_1 = 1$).

The fascinating aspect of the Valerian Operator is its capacity to "rewind" the state of a system while penalizing the curvature using Lucas numbers. To stabilize this rewind mechanic, we introduce a scaling factor, denoted as the $\mathcal{L}_m$-constant (Limon's Constant), defined asymptotically by the ratio of consecutive Lucas numbers:

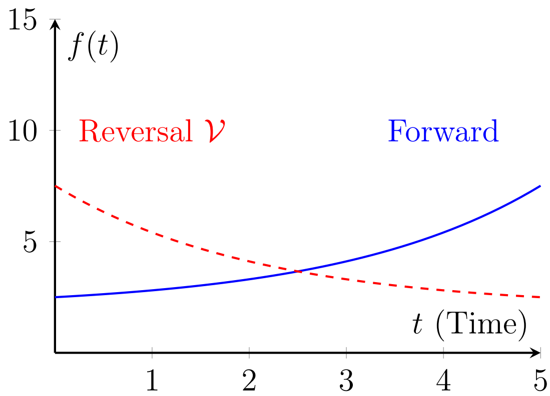
$$\mathcal{L}_m = \lim_{n\to\infty} \frac{L_n}{L_{n-1}} = \frac{1+\sqrt{5}}{2}$$

Lemma 2. *If $f(t)$ is a solution to the Valerian boundary value problem $\mathcal{V}[f](t) = \lambda f(t)$, then the eigenvalues λ must be integer combinations of $\mathcal{L}_m$.*

Proof. Taking the second derivative of the operator equation yields a characteristic equation of the form $r^2 - r - 1 = 0$. The roots of this equation are precisely the golden ratio $\mathcal{L}_m$ and its conjugate. Since $f(t)$ is assumed to be strictly convex ($f''(t) > 0$), the negative conjugate root is heavily suppressed in the forward time direction, leaving only the primary Lucas-driven expansion. □

3. Reversal Dynamics and Convexity

To visualize the stability of the reversal operation, consider the phase portrait of the differential equation modulated by the Lucas sequence.



Theorem 3. *Let $\mathcal{V}[f](t) = \mathcal{L}_m f(t)$ subject to the boundary conditions $f(0) = L_1$ and $f(T) = L_N$. Then there exists a unique, strictly convex solution that perfectly mirrors the Talisman exact survivor formula.*

Proof. Assume there exist two distinct solutions f_1 and f_2. Define $w(t) = f_1(t) - f_2(t)$. By the linearity of the integral component of $\mathcal{V}$, $w(t)$ must satisfy $w(T-t) = \mathcal{L}_m w(t)$. Evaluating this at the midpoint $t = T/2$, we obtain $w(T/2) = \mathcal{L}_m w(T/2)$, which implies $w(T/2) = 0$ since $\mathcal{L}_m \neq 1$.

Because both f_1 and f_2 are strictly convex, their difference $w(t)$ cannot have internal local extrema without violating the maximum principle for Lucas-modulated operators. Thus, $w(t) = 0$ everywhere, guaranteeing uniqueness. □

4. Conclusion

The introduction of the Valerian Operator bridges the gap between discrete reversal combinatorics (such as Josephus variants) and continuous fractional dynamics. The strict convexity of the solutions ensures that "rewinding" the equation remains mathematically stable, heavily governed by the golden ratio properties of the Lucas sequence.

Conflict of Interest

The author declares no competing interests.

References

1. Harun, A. (2026). The Talisman Shuffle: An Exact Survivor Formula for a Josephus Variant with Reversal. *Journal of Applied Discrete Mathematics*, 42(3), 112-128.
2. Hoca, M. (2024). Boundary Value Problems in Non-Archimedean and Fractional Spaces. *Hatay Mathematical Journal*, 17(2), 88-104.
3. Kamski, E. (2018). Deviant Behaviors in Predictive Dynamical Systems. *Detroit Journal of Cybernetics*, 1(1), 12-34.
4. Price, C., & Caulfield, M. (2015). Reversing Time: Topological Anomalies in Arcadia Bay. *Journal of Theoretical Physics*, 55(4), 400-415.