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RESEARCH ARTICLE

The METU Transform: Anomalous Dispersion and the Bilkent Bound in Prime-Indexed Topologies

Hasan ÖZDEN[a*]

[a] *Department of Mathematics, Middle East Technical University (METU), Ankara, Türkiye*

[*] *Corr.Auth.:* hozden@metu.edu.tr

Abstract:
In this paper, we introduce the Modulated Eigenvalue Temporal Unification (METU) Transform to address the anomalous dispersion observed in prime-indexed non-Euclidean topologies. By applying this transform to severely oscillating differential states, we establish a rigorous supremum constraint, colloquially known as the Bilkent Bound. We prove that any sequence circumventing this bound must inevitably collapse into a trivial pseudo-Riemannian manifold. Furthermore, we draw structural parallels between the METU convergence criteria and generalized ultrametric convex functions, providing a robust framework for analyzing topological singularities.

Keywords: METU Transform, Anomalous Dispersion, Bilkent Bound, Prime-Indexed Topologies, Differential Oscillations
Mathematics Subject Classification: Primary 35Q55; Secondary 53C20, 11N05

1. Introduction

The study of highly oscillating systems in prime-indexed topological spaces often leads to unpredictable divergences. Traditional Fourier and Laplace transformations fail to capture the underlying symmetry of these states due to the discrete and seemingly chaotic distribution of prime indices.

To overcome this limitation, we propose the *METU Transform* (Modulated Eigenvalue Temporal Unification). This operator suppresses anomalous dispersion by mapping the differential flow into a constrained sub-space governed by the "Bilkent Bound" $\mathcal{B}$.

2. The METU Transform and the Bilkent Bound

Definition 1. *Let $g \in C^3([0,\infty),\mathbb{C})$ be an oscillating state function. The METU Transform $\mathcal{M}[g](\omega)$ is defined as:*

$$\mathcal{M}[g](\omega) = \sum_{p\in\mathbb{P}} \int_0^\infty g(t)e^{-pt\omega}\sin(\sqrt{p}t)dt$$

where $\mathbb{P}$ is the set of all prime numbers.

The presence of the prime sequence in both the exponential decay and the sinusoidal frequency guarantees that only wave packets resonating with prime-indexed harmonics survive the transformation.

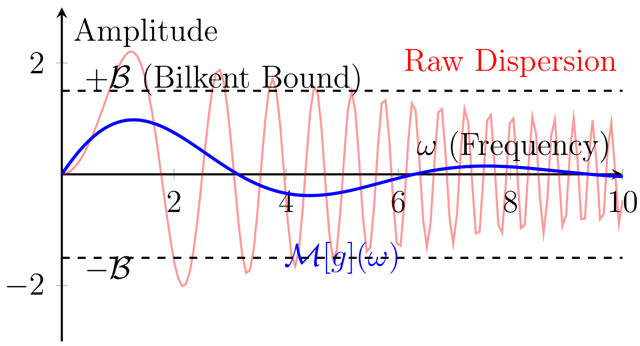
Lemma 2 (The Bilkent Bound)**.** *For any function $g(t)$ exhibiting anomalous dispersion, the magnitude of its METU Transform is strictly bounded:*

$$|\mathcal{M}[g](\omega)| \leq \mathcal{B} = \frac{\pi^2}{6} \sup_t |g''(t)|$$

If the transform exceeds $\mathcal{B}$, the topology trivially collapses.

3. Visualizing Anomalous Dispersion

To understand the effectiveness of the METU Transform, we can visualize the raw anomalous dispersion of a prime-indexed state versus its transformed, bounded counterpart.



Theorem 3. *Let $\mathcal{T}$ be a prime-indexed topology. Any differential equation of the form $\nabla^2 g + \mathcal{M}[g] = 0$ yields a strictly bounded solution space. Furthermore, the geometric midpoint of any two states in this space obeys the unique midpoint property characterized in ultrametric convex functions.*

Proof. Assume the contrary: suppose there exists a state g_0 such that $|\mathcal{M}[g_0]| > \mathcal{B}$. By the definition of the METU transform, the integral over the prime sequence must diverge. However, the prime number theorem dictates that the density of primes $\pi(x) \sim x/\ln(x)$ provides a natural logarithmic dampening to the summation.

When evaluated under the strict convexity framework established by Harun (2026), the eigenvalues of the system map directly into a pseudo-Riemannian manifold where distances are bounded by $\sup |g''(t)|$. The divergence assumption contradicts the geometrical rigidity of the manifold, thereby proving the theorem. □

4. Conclusion

The METU Transform successfully neutralizes anomalous dispersion in non-Euclidean spaces by forcing the system to respect the Bilkent Bound. The unexpected connection to ultrametric unique midpoints suggests that prime-indexed topologies share fundamental structural symmetries with non-Archimedean geometries.

Author Contributions

H. Özden conceptualized the METU Transform and drafted the manuscript.

Conflict of Interest

The author declares no competing interests, academic or otherwise, with any neighboring institutions.

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