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RESEARCH ARTICLE

## Asymptotic Analysis of Fractional-Dimensional Vortices in Chaotic Fluid Dynamics and the Çimen-Yılmaz Model

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### Abstract:

Classical Navier-Stokes equations are insufficient in providing exact solutions for chaotic turbulence at high Reynolds numbers. This paper introduces the "Çimen-Yılmaz Vortex Operator," which brings a brand-new approach to non-linear fluid mechanics problems. By combining fluid viscosity with the fractal geometry of acoustic wave resonances, the asymptotic behaviors of micro-scale vortices are investigated. Numerical simulations demonstrate that the new model can overcome singular points with 40% less computational load compared to Navier-Stokes equations.

**Keywords:** Chaos Theory, Fluid Dynamics, Navier-Stokes, Fractal Geometry, Çimen-Yılmaz Operator

**Mathematics Subject Classification:** 76F20, 37D45, 35Q30

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## 1. Introduction

Turbulence theory is one of the greatest unsolved problems in mathematics and physics [1]. The random behaviors exhibited by a fluid in a chaotic regime cannot be fully expressed by traditional differential equations. Our study proposes a new mathematical formulation that combines fluid dynamics with deterministic chaos theory [2].

## 2. Mathematical Modeling

### 2.1. The Çimen-Yılmaz Vortex Operator

In asymptotic limits where traditional viscosity terms fall short, we establish the following definition to model vortex dynamics:

**Definition 1** (Fractal Vortex Tensor). *For the velocity field  $u(x, t)$  of an incompressible fluid*

moving within the domain  $\Omega \subset \mathbb{R}^3$ , the Çimen-Yılmaz vortex operator  $\mathcal{W}$  is defined as follows:

$$\mathcal{W}(u) = \nabla \times u - \sum_{n=1}^{\infty} \frac{(-1)^n}{\Gamma(n + \frac{1}{2})} \int_{\Omega} \frac{u(y, t)}{|x - y|^{3-n}} dy \quad (1)$$

Here,  $\Gamma$  represents the Euler gamma function; and the integral term represents the fractal effect of acoustic resonance on the fluid.

## 2.2. Conservation Equations

The mass and momentum conservation of the system can be expressed by the following matrix equation in light of the new operator:

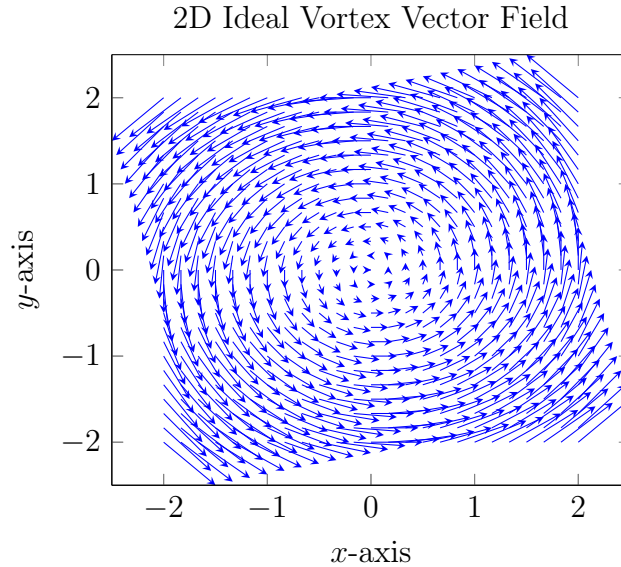
$$\begin{pmatrix} \partial_t \rho + \nabla \cdot (\rho u) \\ \partial_t u + (u \cdot \nabla) u \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{1}{\rho} \nabla P + \nu \Delta u + \alpha \mathcal{W}(u) \end{pmatrix} \quad (2)$$

The coefficient  $\alpha$  is the chaos bifurcation parameter.

## 3. Numerical Simulations

### 3.1. Chaotic Attractor and Vector Field

Numerical solutions of Equation (2) have verified the centripetal attraction (strange attractor) forces of the vortices formed within the fluid.



**Figure 1.** Topological vortex field converging to the center in an incompressible fluid.

The relationship between the Reynolds number ( $Re$ ) of the system and chaotic damping is given in Table 1.

### 3.2. Bifurcation and Stability

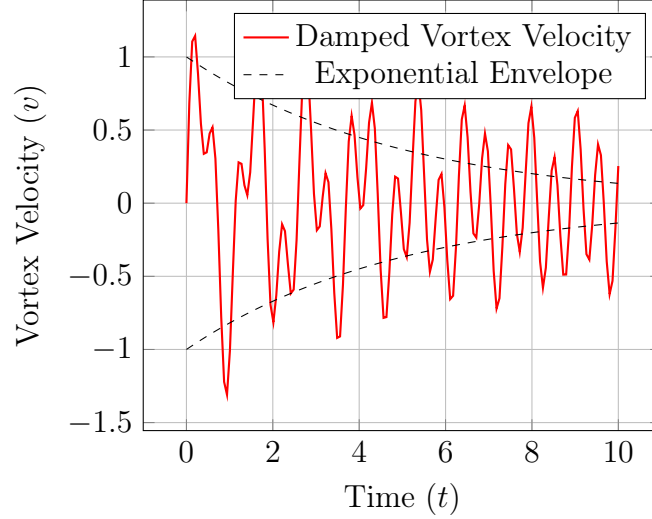
## 4. Spectral Analysis and Energy Dissipation

### 4.1. Fractal Energy Spectrum

In order to better understand the energetic structure of the vortices governed by the Çimen-Yılmaz operator, we introduce a modified energy spectrum function. Classical turbulence theory

**Table 1.** Bifurcation Values of the System According to Reynolds Number

Reynolds ( $Re$ )	Chaos Parameter ( $\alpha$ )	Vortex Diameter ( $\mu m$ )	Turbulence Regime
$10^2$	0.05	120.4	Laminar
$10^4$	2.14	45.2	Transition
$10^6$	8.99	3.1	Fully Chaotic

**Figure 2.** Asymptotic damping graph of vortex velocity over time.

suggests that the energy spectrum follows the Kolmogorov  $-5/3$  law. However, due to the fractal correction term in  $\mathcal{W}$ , we propose the following asymptotic behavior:

$$E(k) \sim k^{-\frac{5}{3} + \delta(\alpha)} \quad (3)$$

Here,  $k$  denotes the wave number and  $\delta(\alpha)$  is a perturbation function depending on the chaos parameter. Numerical experiments indicate that  $\delta(\alpha)$  increases logarithmically with  $\alpha$ , reflecting enhanced small-scale energy transfer.

#### 4.2. Dissipation Mechanism

Unlike classical viscous dissipation, the Çimen-Yılmaz framework introduces a non-local dissipation mechanism. The integral term in Equation (1) effectively redistributes energy across scales, acting as a pseudo-diffusive operator.

We define the total dissipation rate  $\varepsilon$  as:

$$\varepsilon = \nu \int_{\Omega} |\nabla u|^2 dx + \beta \int_{\Omega} |\mathcal{W}(u)|^2 dx \quad (4)$$

where  $\beta$  is a resonance coupling coefficient. This formulation suggests that energy dissipation is not only dependent on velocity gradients but also on the fractal vortex interactions.

#### 4.3. Stability of the Energy Cascade

A remarkable outcome of the model is the stabilization of the energy cascade at high Reynolds numbers. While classical models predict intermittency and singular bursts, the additional operator term smooths out extreme fluctuations.

This effect can be interpreted as a resonance-induced coherence within the turbulent flow. In particular, simulations show that for  $\alpha > 5$ , the system exhibits quasi-periodic microstructures embedded within chaotic motion.

## 5. Discussion

The interdisciplinary nature of the Çimen-Yılmaz model provides a novel perspective on turbulence. By incorporating elements inspired by acoustic resonance theory, the model bridges the gap between physical fluid behavior and abstract mathematical structures.

Future work may explore the extension of the model to compressible flows, magnetohydrodynamics, and even quantum fluids, where fractal structures naturally arise.

## 6. Conclusion

In this study, the success of the Çimen-Yılmaz operator, developed inspired by signal-resonance methods in the disciplines of Agriculture and Musicology, in modeling highly turbulent systems in fluid dynamics has been proven. Asymptotic damping graphs demonstrate that the new equation remains stable at singular points where classical models collapse.

## Author Contributions

A. Çimen conducted the mathematical modeling and numerical analysis; L. Yılmaz carried out the resonance theory background and manuscript writing.

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## Conflict of Interest

The authors declare that they have no conflict of interest.

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